

# Discussion of a possible corrected black hole entropy

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**ABSTRACT:** Einstein's equation could be interpreted as the first law of thermodynamic near the spherically symmetric horizon. By using this method, we investigate the Eddington-inspired Born-Infeld (EiBI) gravity. Without matter field, the EiBI gravity can also derive the first law. With an electromagnetic field, as the field equations have a more general spherically symmetric solution in EiBI gravity, we find that the entropy would have a correction. Through recalling the Einstein gravity with a more general static spherical symmetric, this correction of the entropy might be generalized to Einstein gravity. Furthermore, we point out that the Einstein gravity and EiBI gravity might be equivalent on the event horizon. At last, under EiBI gravity with the electromagnetic field, a specific corrected entropy of black hole is given.

**KEYWORDS:** Black Holes, Thermodynamics, Other Theories of Gravity

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## 1 Introduction

Black hole thermodynamics has been proposed for many years since the entropy and temperature were found by Bekenstein and Hawking [1, 2], even got many interesting results, like the four laws of black hole thermodynamics. It established the connection between the gravity and thermodynamics.

The entropy is assumed to be proportional to its horizon area [1], also it is well-known that the so-called area formula of black hole entropy holds only in Einstein gravity. However, when some higher order curvature term appears in some gravity theory, the area formula has to be modified [3]. A logarithmic term often occurs in the correction like the black hole entropy in loop quantum gravity (quantum geometry) [4–6] and thermal equilibrium fluctuation [7, 8]. Even in other gravity theory the correction of entropy has been studied like Gauss-Bonnet gravity [9], Lovelock gravity [10] and  $f(R)$  gravity [11]. In the apparent horizon of FRW universe, the entropy also has a correction [12].

The Einstein's equation can be derived from the thermodynamics [13], on the other side, the thermodynamic route to the gravity field equation, which could get the first law of thermodynamic in Einstein gravity, was proposed by T. Padmanabhan [14]. It suggested a generic connection between thermodynamics of horizons and gravity, although it's not yet understood at a deeper level [15]. This technique has been used in Gauss-Bonnet gravity and Lanczos-Lovelock gravity [16], the corrected entropy is the same in [17, 18], respectively.

The EiBI gravity was inspired by Banados and Ferreira [19]. It is completely equivalent to the Einstein gravity in vacuum, but in the presence of matter, it would show many interesting results, like an alternative theory of Big Bang singularity in early universe [20] and the mass inflation in EiBI black holes [21, 22]. However, there are few investigation for the thermodynamic properties of black hole in EiBI gravity, as it has a complicated spherically symmetric solution with the electromagnetic field [23].

In this paper, we derive the first law of black hole thermodynamics from the Einstein's equation near the event horizon, which is analogous with the technique proposed by T. Padmanabhan [14, 16]. we also use this technique in EiBI gravity and get the known first law, the results show a more general formula of entropy, which also holds for Ads Schwarzschild black hole and Ads R-N black hole. Motived by this, by supposing a more general static spherically symmetric, we also get the same result in Einstein gravity.

This paper is organized as follows: In Sec.II, there is a derivation of the thermodynamic identity from Einstein gravity. In Sec.III, we investigate the EiBI gravity by using the thermodynamic route to the field equation, and get the formula of entropy. In Sec.IV, the same result has been got by recalling the Einstein gravity for a more general static spherically symmetric. In Sec.V, a corrected entropy in EiBI gravity with electromagnetic field is given. Conclusions and discussions are given in Sec.VI.

## 2 Black hole thermodynamic identity from Einstein's equation

The Einstein's equation can be derived from the thermodynamics [13]. On the other side, it is possible to interpret the Einstein's equation near the spherical symmetric event horizon as the first law of thermodynamics. Considering a static spherically symmetric spacetime

$$ds^2 = -f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad , \quad (2.1)$$

and the event horizon  $r = r_H$  satisfying  $f(r_H) = 0$ , then one can get its thermodynamic quantities

$$T = \frac{\kappa}{2\pi} = \frac{f'(r_H)}{4\pi}, \quad S = \pi r_H^2, \quad V = \frac{4\pi}{3}r_H^3. \quad (2.2)$$

If we consider a Anti-de-Sitter spacetime with a negative cosmological constant  $\Lambda$ , there would be a pressure  $P = -\Lambda/8\pi$  [24]. The mass of black hole was treated as enthalpy and the first law of black hole thermodynamics is

$$dH = dM = TdS + VdP \quad . \quad (2.3)$$

Through the Legendre transformation, one can get

$$dU = TdS - PdV \quad . \quad (2.4)$$

The Einstein's equation with a cosmological constant is

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu} \quad . \quad (2.5)$$

If the metric has the form of eq.(2.1), in the case of  $T_{\mu\nu} = 0$ , one can obtain the  $\theta\theta$  component of the equation

$$-1 + f(r) + rf'(r) = -\Lambda r^2 \quad , \quad (2.6)$$

Setting  $r = r_H$ , then multiplying above equation by  $dr_H$ , we can rewrite eq.(2.6) as

$$\frac{f'(r_H)}{4\pi}d(\pi r_H^2) - d\left(\frac{r_H}{2}\right) = -\frac{\Lambda}{8\pi}d\left(\frac{4\pi r_H^3}{3}\right) \quad . \quad (2.7)$$

Noticing eq.(2.2), above equation can be regarded as the first law of black hole thermodynamics, since  $U = r_H/2$  for the Schwarzschild solution.

For a charged Ads black hole, the metric also takes the form of eq.(2.1). The energy-momentum tensor of electromagnetic field is

$$T_{\mu\nu} = \frac{1}{4\pi}(F_{\mu\sigma}F_{\nu}^{\sigma} - \frac{1}{4}g_{\mu\nu}F_{\sigma\rho}F^{\sigma\rho}) \quad . \quad (2.8)$$

Its none zero components are:

$$\begin{aligned} T_{tt} &= fE_0^2/8\pi, & T_{rr} &= -f^{-1}E_0^2/8\pi, \\ T_{\theta\theta} &= r^2E_0^2/8\pi, & T_{\phi\phi} &= r^2\sin^2\theta E_0^2/8\pi, \end{aligned}$$

where  $E_0 = Q/r^2$ , and  $Q$  represents the charge of black hole. According to the Einstein's equations, one can also get

$$-1 + f(r) + rf'(r) = -\Lambda r^2 - r^2E_0^2 \quad . \quad (2.9)$$

By the same technique, and treating  $Q$  as a constant, we get

$$\frac{f'(r_H)}{4\pi}d(\pi r_H^2) - d\left(\frac{r_H}{2} + \frac{Q^2}{2r_H}\right) = Pd\left(\frac{4\pi r_H^3}{3}\right) \quad , \quad (2.10)$$

which can be also treat as the first law with the thermodynamic quantities in eq.(2.2), and one can verify that  $U = r_H/2 + Q^2/2r_H$  for the Ads R-N black hole.

Here we keep  $Q$  as a constant, which means a chargeless particle falls into the Ads R-N black hole, eq.(2.10) is consist with the first law. For a charged particle falls into the Ads R-N black hole, the event horizon  $r_H$  would arise due to changes of  $dM$  and  $dQ$ , then eq.(2.10) could be rewrite as [16]

$$\frac{f'(r_H)}{4\pi}d(\pi r_H^2) - d\left(\frac{r_H}{2} + \frac{Q^2}{2r_H}\right) + \frac{Q}{r_H}dQ = Pd\left(\frac{4\pi r_H^3}{3}\right) \quad . \quad (2.11)$$

Then it would adopt to the first law with the formulation

$$dU = TdS - PdV + \Phi dQ \quad . \quad (2.12)$$

We should point out that the  $T_{\mu\nu}$  contributes to the internal energy  $U$ .

Thus the Einstein's equation can be interpreted as the first law of thermodynamic near the event horizon. The analogous technique was first proposed by T. Padmanabha, and some relevant comments about the meaning of thermodynamic quantities for this result were given [14–16, 25]. Note that the structure of the equation itself allows us to read off the expression for entropy. This technique has been used for Gauss-Bonnet gravity and Lovelock gravity [16], in which their entropy expressions are the same with [17, 18], respectively. In the next part, we will show our discussion of this method in the Eddington-inspired Born-Infeld gravity [19].

### 3 The entropy in Eddington-inspired Born-Infeld gravity

The Eddington-inspired Born-Infeld theory of gravity is based on the Palatini formulation which treats the metric and connection as independent fields [19]. Its action can be written as

$$S = \frac{1}{8\pi\kappa} \int d^4x [\sqrt{|g_{\mu\nu} + \kappa R_{\mu\nu}(\Gamma)|} - \lambda\sqrt{g}] + S_M(g, \Gamma, \Psi) \quad , \quad (3.1)$$

where  $g_{\mu\nu}$  is the metric of spacetime and its determinant is  $g$ ,  $R_{\mu\nu}$  is the symmetric Ricci tensor related to  $\Gamma$ , the dimensionless parameter  $\lambda = 1 + \kappa\Lambda$ , and the parameter  $\kappa$  has the inverse dimension of cosmological constant  $\Lambda$ .

By varying the action with respect to  $g_{\mu\nu}$  and  $\Gamma$ , one obtains the equation of motion

$$q_{\mu\nu} = g_{\mu\nu} + \kappa R_{\mu\nu} \quad , \quad (3.2)$$

$$\sqrt{|q|}q^{\mu\nu} = \lambda\sqrt{|g|}g^{\mu\nu} - 8\pi\kappa\sqrt{|g|}T^{\mu\nu} \quad , \quad (3.3)$$

where  $q_{\mu\nu}$  is the auxiliary metric compatible to the connection with

$$\Gamma_{\mu\nu}^\lambda = \frac{1}{2}q^{\lambda\sigma}(q_{\mu\sigma,\nu} + q_{\nu\sigma,\mu} - q_{\mu\nu,\sigma}) \quad . \quad (3.4)$$

By combining eq.(3.2) and eq.(3.3), then expanding the field equations to 2nd order of  $\kappa$  [19]

$$R_{\mu\nu} \simeq \Lambda g_{\mu\nu} + 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}) + 8\pi\kappa[S_{\mu\nu} - \frac{1}{4}Sg_{\mu\nu}] \quad , \quad (3.5)$$

where  $S_{\mu\nu} = T_\mu^\alpha T_{\alpha\nu} - \frac{1}{2}TT_{\mu\nu}$ , one can find that the equation is the 1st order corrections to Einstein' equation. On the other hand, EiBI gravity can be interpreted as a correction of the matter term compared with Einstein gravity. Even the EiBI gravity is fully equivalent to the Einstein gravity in vacuum.

Let's consider the thermodynamics from the field equation in this gravity model. Generally, a static spherically symmetric for  $g_{\mu\nu}$  could be

$$ds_g^2 = -\psi^2(r)f(r)dt^2 + \frac{1}{f(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \quad , \quad (3.6)$$

and the auxiliary metric  $q_{\mu\nu}$  is assumed as [23]

$$ds_q^2 = -G^2(r)F(r)dt^2 + \frac{1}{F(r)}dr^2 + H^2(r)(d\theta^2 + \sin^2\theta d\phi^2) \quad . \quad (3.7)$$

The Ricci tensor was calculated as below

$$R_{tt} = 2\frac{GG'H'F^2}{H} + \frac{G^2FF'H'}{H} + \frac{3}{2}GG'FF' + GG''F^2 + \frac{1}{2}G^2FF'' \quad , \quad (3.8)$$

$$R_{rr} = -2\frac{H''}{H} - \frac{F'H'}{FH} - \frac{3}{2}\frac{G'F'}{GF} - \frac{G''}{G} - \frac{F''}{2F} \quad , \quad (3.9)$$

$$R_{\theta\theta} = 1 - HH'F' - \frac{G'}{G}HH'F - H'^2F - HH''F \quad , \quad (3.10)$$

$$R_{\phi\phi} = \sin^2\theta R_{\theta\theta} \quad , \quad (3.11)$$

where the  $X' = \partial X / \partial r$ , which we will use in the rest of this paper.

Without the matter fields, the eq.(3.3) reduces to

$$\frac{H^2}{GF} = \frac{\lambda r^2}{\psi f} \quad , \quad (3.12)$$

$$GH^2F = \lambda r^2 \psi f \quad , \quad (3.13)$$

$$G = \lambda \psi \quad , \quad (3.14)$$

thus one can obtain

$$G = \lambda \psi, \quad F = \lambda^{-1} f, \quad H^2 = \lambda r^2. \quad (3.15)$$

Plugging these into eq.(3.2), then the  $\theta\theta$  component is

$$1 - r f'(r) - \lambda f(r) = \frac{1}{\kappa} (\lambda - 1) r^2 \quad . \quad (3.16)$$

Near the event horizon  $r = r_H$  ( $f(r_H) = 0$ ), we have

$$d\left(\frac{r_H}{2}\right) - \frac{f'(r_H)}{4\pi} d(\pi r_H^2) = -P d\left(\frac{4\pi r_H^3}{3}\right) \quad . \quad (3.17)$$

As the EiBI gravity is fully equivalent to the Einstein gravity in vacuum [19], the above equation could imply the first law. In fact, the black hole solution to EiBI gravity with no source is the same as Schwarzschild-de Sitter metric, which illustrates  $\psi = 1$  and  $f = 1 - 2m/r + \Lambda r^2/3$  [19]. The thermodynamic quantities of eq.(2.2) also hold in EiBI gravity here in eq.(3.17). Thus the first law can also be got from the EiBI gravity. Next, we would consider the EiBI gravity with matter field.

With the electromagnetic field, One can get the energy-momentum tensor according to eq.(2.8) and eq.(3.6)

$$\begin{aligned} T^{tt} &= (\psi^2 f)^{-1} E_0^2 / 8\pi, & T^{rr} &= -f E_0^2 / 8\pi, \\ T^{\theta\theta} &= r^{-2} E_0^2 / 8\pi, & T^{\phi\phi} &= r^{-2} \sin^{-2} \theta E_0^2 / 8\pi, \end{aligned}$$

where  $E_0 = Q/r^2$  and the  $Q$  represents the charge of black hole. Then the eq.(3.3) becomes

$$\frac{H^2}{GF} = (\lambda + \kappa E_0^2) \frac{r^2}{\psi f} \quad , \quad (3.18)$$

$$GH^2F = (\lambda + \kappa E_0^2) r^2 \psi f \quad , \quad (3.19)$$

$$G = (\lambda - \kappa E_0^2) \psi \quad , \quad (3.20)$$

and one can get

$$G = (\lambda - \kappa E_0^2) \psi, \quad F = (\lambda - \kappa E_0^2)^{-1} f, \quad H^2 = (\lambda + \kappa E_0^2) r^2. \quad (3.21)$$

The  $\theta\theta$  component of eq.(3.2) is written as

$$1 - \frac{\lambda + \kappa E_0^2 + \kappa r E_0 E_0'}{\lambda - \kappa E_0^2} \cdot r f' - \frac{Y}{\lambda - \kappa E_0^2} f = \frac{1}{\kappa} (\lambda - 1) r^2 + E_0^2 r^2 \quad , \quad (3.22)$$

where

$$Y = 2\kappa r E_0 E'_0 + \frac{G'}{G} H H' - H'^2 \quad . \quad (3.23)$$

If we assume that the event horizon satisfies  $f(r_H) = 0$  and  $\psi(r_H) \neq 0$ , then set  $r = r_H$  in eq.(3.22) and multiply it by  $dr_H$ , noting  $E'_0 = -2E_0/r$ , it gives

$$d\left(\frac{r_H}{2} + \frac{Q^2}{2r_H}\right) - \frac{r_H}{2} f'(r_H) dr_H = -P d\left(\frac{4\pi r_H^3}{3}\right) \quad . \quad (3.24)$$

This equation should be the first law since it can go back to eq.(3.17) when  $Q = 0$ . And one can also confirm this by noticing that  $dU = d(r_H/2 + Q^2/2r_H)$  and  $dV = d(4\pi r_H^3/3)$ . Moreover, it gives the same result in Einstein gravity eq.(2.10). So we should identify that

$$TdS = \frac{r_H}{2} f'(r_H) dr_H \quad . \quad (3.25)$$

However, as the metric takes the form of eq.(3.6), the surface gravity could be [26]:

$$\kappa = \lim_{r \rightarrow r_H} \frac{1}{2} \frac{\partial_r g_{tt}}{\sqrt{g_{tt} g_{rr}}} = \frac{\psi(r_H) f'(r_H)}{2} \quad . \quad (3.26)$$

One can obtain the temperature on the event horizon

$$T = \frac{\psi(r_H) f'(r_H)}{4\pi} \quad . \quad (3.27)$$

Then eq.(3.25) would imply that

$$dS = \left(\frac{2\pi r_H}{\psi(r_H)}\right) dr_H \quad , \quad (3.28)$$

or

$$S = \int \frac{2\pi r_H}{\psi(r_H)} dr_H \quad . \quad (3.29)$$

Obviously, when  $\psi = 1$ , one can get  $S = \pi r_H^2$ .

Above all, it seems that the entropy eq.(3.29) is just for the EiBI gravity. However, we will show that it also holds in Einstein gravity once metric takes the form of eq.(3.6).

## 4 The entropy in Einstein gravity

In the Einstein gravity, we consider a more general static spherical symmetric configurations of the form

$$ds^2 = -\psi(r)^2 f(r) dt^2 + \frac{1}{f(r)} dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad . \quad (4.1)$$

One can calculate its nonzero components of Ricci tensor

$$R_{tt} = \frac{(\psi^2 f)'' f}{2} - \frac{(\psi^2 f)' f}{4} \left(-\frac{f'}{f} + \frac{(\psi^2 f)'}{\psi^2 f}\right) + \frac{(\psi^2 f)' f}{r} \quad , \quad (4.2)$$

$$R_{rr} = -\frac{(\psi^2 f)''}{2\psi^2 f} + \frac{(\psi^2 f)'}{4\psi^2 f} \left(-\frac{f'}{f} + \frac{(\psi^2 f)'}{\psi^2 f}\right) - \frac{f'}{rf} \quad , \quad (4.3)$$

$$R_{\theta\theta} = 1 - \frac{rf}{2} \left(\frac{f'}{f} + \frac{(\psi^2 f)'}{\psi^2 f}\right) - f \quad , \quad (4.4)$$

$$R_{\phi\phi} = \sin^2 \theta R_{\theta\theta} \quad . \quad (4.5)$$

And the Einstein's equation with a cosmological constant can be written as

$$R_{\mu\nu} - \Lambda g_{\mu\nu} = 8\pi(T_{\mu\nu} - \frac{1}{2}Tg_{\mu\nu}) \quad . \quad (4.6)$$

For the Schwarzschild vacuum  $T_{\mu\nu} = 0$ , according to eq.(4.6), eq.(4.2) and eq.(4.3) one can get  $\psi = C$ , a constant. We can always have  $\psi = 1$  by choosing the coordinate time  $d\hat{t} = Cdt$  without changing the killing vector, then the metric would back to the spherically symmetric eq.(2.1). For the charged black hole, we have got the Reissner-Nordstrom metric, which implies  $\psi = 1$ , thus the first law could be obtained. The reason of this result might be that the matter field leads to  $\psi = 1$ , according to the Einstein's equation.

To get a general case, we just consider the  $\theta\theta$  component of eq.(4.6), which can be expressed as

$$1 - \frac{rf(r)}{2} \left( \frac{f'(r)}{f(r)} + \frac{(\psi^2 f(r))'}{\psi^2 f(r)} \right) - f(r) = \Lambda r^2 + 8\pi(T_{\theta\theta} - \frac{1}{2}Tr^2) \quad . \quad (4.7)$$

We assume the metric satisfies  $f(r_H) = 0$  and  $\psi(r_H) \neq 0$  on the event horizon. By setting  $r = r_H$  and considering the matter field contributes to the internal energy  $U$ , then multiplying  $dr_H$  one can get

$$dU - \frac{r_H}{2} f'(r_H) dr_H = -Pd \left( \frac{4\pi r_H^3}{3} \right) \quad . \quad (4.8)$$

There is nothing different from eq.(3.24) which was got from the EiBI gravity. It implies that the Einstein gravity and EiBI gravity might be equivalent on the event horizon from the view of black hole thermodynamics.

The Hawking temperature on the event horizon becomes

$$T = \frac{\kappa}{2\pi} = \frac{\psi(r_H) f'(r_H)}{4\pi} \quad . \quad (4.9)$$

Now rewrite eq.(4.8) as

$$dU - T \left( \frac{2\pi r_H}{\psi(r_H)} dr_H \right) = -PdV \quad . \quad (4.10)$$

One would find the entropy has to satisfy

$$dS = \frac{2\pi r_H}{\psi(r_H)} dr_H \quad , \quad (4.11)$$

or

$$S = \int \frac{2\pi r_H}{\psi(r_H)} dr_H \quad , \quad (4.12)$$

once  $\psi = 1$ , it is obvious that  $S = \pi r_H^2 = A/4$ , which is the well-known Bekenstein-Hawking black hole entropy. For the Schwarzschild black hole and R-N black hole, which all have  $\psi = 1$ ,  $S = A/4$ . However, if there is a static spherically symmetric solution taking the form of eq.(4.1), we may conjecture that the entropy would be amended. In fact, the "dirty" black hole could have the spherical symmetric like eq.(4.1) [26]. Moreover, in ref [27, 28], it has been shown that the entropy of the "dirty" black hole should be corrected with some terms of integrals over the event horizon.



## 5 A corrected entropy in Eddington-inspired Born-Infeld gravity

Now, let's return back to the EiBI gravity, as it has the solution taking the form of eq.(3.6). The black hole solution with electromagnetic field in EiBI gravity has been found, while  $f(r_H) = 0$  and  $\psi(r_H) \neq 0$  [19, 23]. It is given as below

$$\psi(r) = \frac{\sqrt{\lambda} r^2}{\sqrt{\lambda r^4 + \kappa Q^2}} \quad . \quad (5.1)$$

With this result we can get the corrected entropy

$$S = \int \frac{2\pi r_H}{\psi(r_H)} dr_H = \pi \int \frac{1}{r_H^2} \sqrt{r_H^4 + \frac{\kappa}{\lambda} Q^2} dr_H^2 \quad (5.2)$$

$$= \pi \sqrt{r_H^4 + \frac{\kappa}{\lambda} Q^2} - \pi \sqrt{\frac{\kappa}{\lambda}} |Q| \cdot \ln \left( \sqrt{\frac{\kappa}{\lambda}} \frac{|Q|}{r_H^2} + \sqrt{1 + \frac{\kappa}{\lambda} \frac{Q^2}{r_H^4}} \right) \quad . \quad (5.3)$$

A logarithmic term occurs in this formula as a corrected entropy. When  $Q = 0$ , one gets  $S = \pi r_H^2$ , which is consistent with the vacuum case, and so is the same as Einstein gravity. When  $\kappa \rightarrow 0$ , EiBI gravity would reduce to the Einstein gravity, and entropy becomes the Bekenstein-Hawking one. Moreover, the logarithmic term in the corrected entropy has been proposed from the loop quantum gravity [6], and conformal anomaly theory [29]. Even the corrected entropy-area relation for apparent horizon in FRW universe takes the similar form [12].

## 6 Conclusion and discussions

In Summary, in this paper, we started with the Einstein's equation and show that the first law of black hole thermodynamic could be obtained near the event horizon for a static spherically symmetric. The analogous technique was proposed by T.Padmanabhan [14–16]. Since it provided a convenient approach to study the black hole thermodynamics just from the field equation, we investigated the black hole thermodynamic in EiBI gravity. We found that there is nothing different from Einstein gravity in vacuum, but entropy could be different from the R-N black hole when considering the electromagnetic field. The corrected entropy from EiBI gravity should be eq.(3.29), which can reduce to the Bekenstein-Hawking entropy when  $\psi = 1$ , and it is also the same as Einstein gravity when  $Q = 0$  without the matter field [19].

Next, we investigated a more general static spherically symmetric taking the form eq.(4.1) in Einstein gravity, due to the solution of EiBI gravity has the same form. It is surprised to found that the Einstein gravity implied the same result, which means that the entropy might also have a correction taking the form of eq.(4.12). Thus the correction of entropy from EiBI might also hold in the Einstein gravity for a static spherically symmetric like eq.(4.1).

Moreover, as the Einstein gravity and EiBI gravity hold the same result, we remarked that these two theories of gravity could be equivalent on the event horizon from the view of thermodynamics.

At last, as an example, a specific corrected entropy of the charged black hole in EiBI gravity was given. The logarithmic term in our corrected entropy is analogous to the theory of loop quantum gravity [4–6] and conformal anomaly theory [29].

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